Exotic Derivatives & Structured Products

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Part 1: Exotic Derivatives

- Over the counter products
- Generally more profitable (and more risky) than vanilla derivatives
- Why do they exist? Possible reasons:
  - To meet special hedging needs.
  - Might be attractive for regulatory, accounting, tax or legal reasons.
  - To reflect a particular view on the market.
  - To appear more attractive than a vanilla product (thus it can be overpriced).
Packages

• A portfolio of:
  – vanilla European call/put options
  – forward contracts
  – the underlying asset
  – Cash

• Examples: bull spreads, bear spreads, butterfly spreads, calendar spreads, straddles, strangles (see next slide).

• Can be set up to have zero initial cost
Packages - Examples

<table>
<thead>
<tr>
<th>Packages</th>
<th>Diagram 1</th>
<th>Diagram 2</th>
<th>Diagram 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bull spread</td>
<td><img src="bullspread.png" alt="Diagram" /></td>
<td><img src="bearspread.png" alt="Diagram" /></td>
<td><img src="butterfliespread.png" alt="Diagram" /></td>
</tr>
<tr>
<td>Bear spread</td>
<td><img src="bullspread.png" alt="Diagram" /></td>
<td><img src="bearspread.png" alt="Diagram" /></td>
<td><img src="butterfliespread.png" alt="Diagram" /></td>
</tr>
<tr>
<td>Butterfly spread</td>
<td><img src="bullspread.png" alt="Diagram" /></td>
<td><img src="bearspread.png" alt="Diagram" /></td>
<td><img src="butterfliespread.png" alt="Diagram" /></td>
</tr>
<tr>
<td>Calendar spread</td>
<td><img src="bullspread.png" alt="Diagram" /></td>
<td><img src="bearspread.png" alt="Diagram" /></td>
<td><img src="butterfliespread.png" alt="Diagram" /></td>
</tr>
<tr>
<td>Straddle (short)</td>
<td><img src="bullspread.png" alt="Diagram" /></td>
<td><img src="bearspread.png" alt="Diagram" /></td>
<td><img src="butterfliespread.png" alt="Diagram" /></td>
</tr>
<tr>
<td>Strangle (long)</td>
<td><img src="bullspread.png" alt="Diagram" /></td>
<td><img src="bearspread.png" alt="Diagram" /></td>
<td><img src="butterfliespread.png" alt="Diagram" /></td>
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</tbody>
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Nonstandard American Options

Variations compared to standard American options:

- No expiry date: **perpetual option** (priced analytically).
- Early exercise is restricted to specific dates: **Bermudan option**.
- Early exercise is restricted to specific time period (e.g. warrant).
- Strike price may change in time (e.g. warrant).
Nonstandard European Options

• **Gap option**: European call which pays off $S_T - K'$ if $S_T > K$ (priced analytically).

• **Shout option**: European option with additional optionality: the holder can “shout” once before maturity, and the option’s intrinsic value at that time is recorded. At maturity the holder receives the maximum of the standard payoff or the recorded intrinsic value. Pricing is similar to American options.
Forward Start Options

- **Forward start option**: standard option but starts at a future time T. Strike can be specified as at-the-money, i.e. the underlying’s price at T.
- **Cliquet option**: A series of forward start options which start at regular intervals and whose strikes are determined by some rule (e.g. the at-the-money rule).
Compound and Chooser Options

• **Compound option**: option on an option. Four main types: call/put on a call/put. Two strikes/ expiry dates, one for the option and one for the underlying option. European option on a European option can be priced analytically.

• **Chooser option**: the holder can choose at a fixed time (before expiry) whether it is a call or a put. Useful when a large price fluctuation is expected but the direction is uncertain, e.g. the stock of a biotech company before the clinical tests of its key product. European-style can be priced analytically.
Barrier Options

The payoff depends on if the underlying’s price breaches barrier H in a specified time period. Main types:

• Knock-in: the option can be exercised only if the barrier is breached.
• Knock-out: the option cannot be exercised if the barrier is breached.

Variations: barrier breach is checked at a fixed time every day; underlying’s price must be above/below barrier for a certain time to knock-in/out the option; double barrier.
Binary (Digital) Options

Pays off some fixed cash (cash-or-nothing) or the asset (asset-or-nothing) if the asset’s price is above/below (call/put) the strike price at expiry. European-style binary options can be priced analytically.
Lookback Options

The payoff depends on the underlying’s minimum/maximum price during the life of the option. Payoff for various types:

- Floating lookback call: \( \max(0, S_T - S_{\min}) \)
- Floating lookback put: \( \max(0, S_{\max} - S_T) \)
- Fixed lookback call: \( \max(0, S_{\max} - K) \)
- Fixed lookback put: \( \max(0, K - S_{\min}) \)
Asian Options

The payoff depends on the underlying’s arithmetic/geometric average price during a specified time period, $S_{avg}$. Payoff for various types:

- Average price call: $\max(0, S_{avg} - K)$
- Average price put: $\max(0, K - S_{avg})$
- Average strike call: $\max(0, S_T - S_{avg})$
- Average strike put: $\max(0, S_{avg} - S_T)$
Volatility Swaps

Exchange the underlying’s realized volatility, $\hat{\sigma}$, for a fixed volatility, $\sigma_K$, at a future time $T$:

$$\text{Payoff}_{\text{vol.\ swap}} = N (\hat{\sigma} - \sigma_K),$$

where $N$ is the notional, and the realized volatility for $n$ daily observations preceding time $T$ is

$$\hat{\sigma} = \sqrt{\frac{252}{n-2} \sum_{i=1}^{n-1} \left[ \ln \left( \frac{S_{i+1}}{S_i} \right) \right]^2}$$

where $S_i$ is the $i^{th}$ observation.

In contrast to options, volatility swaps have exposure only to volatility.
Variance Swaps

Same as volatility swaps, but the underlying’s realized variance, $\mathcal{V}$, is exchanged for a fixed variance rate, $V_K$, at a future time $T$:

$$\text{Payoff}_{\text{var. swap}} = N (\mathcal{V} - V_K),$$

where $\mathcal{V} = \sigma^2$. 
Part 2: Structured Products

Structured products are pre-packaged investment vehicles based on:

- equities, baskets of equities, equity indexes,
- commodities, commodity futures,
- interest rates,
- FX rates,
- debt securities,
- other assets or derivatives.

Structured products are created by financial institutions to meet specific investment needs that usually cannot be met by the standardized financial instruments available in the markets.
Uncapped/Capped Capital Protection

Market expectations:
- Rising underlying
- Sharply falling underlying possible

Characteristics:
- Minimum redemption at expiry equivalent to the capital protection
- Value may fall below capital protection during the lifetime
- Coupon payment possible
- Unlimited participation in a positive performance of the underlying above the Strike (Uncapped only)
- Participation in a positive performance of the underlying up to the Cap (Capped only)
- Limited profit potential (Capped only)
Capital Protection with Knock-out

Market expectations:
- Rising underlying
- Sharply falling underlying possible

Characteristics:
- Minimum redemption at expiry equivalent to the capital protection
- Value may fall below capital protection during the lifetime
- Participation in a positive performance of the underlying until Knock-Out
- Possible payment of a Rebate following a Knock-Out
- Limited profit potential
Capital Protection with Coupon

Market expectations:
- Rising underlying
- Sharply falling underlying possible

Characteristics:
- Minimum redemption at expiry equivalent to the capital protection
- Value may fall below capital protection during the lifetime
- The coupon amount is dependent on the development of the underlying
- Limited profit potential
Discount Certificates

**Market expectations:**
- Underlying moving sideways or slightly rising
- Falling volatility

**Characteristics:**
- Should the underlying close below the Strike at expiry, the underlying and/or a cash amount is redeemed
- Discount Certificates enable investors to acquire the underlying at a lower price.
- Corresponds to a buy-write strategy
- Reduced loss potential compared to a direct investment
- Limited profit potential

**Buy-write strategy:** an investment strategy in which the investor buys stocks and writes call options against the stock position.
Barrier Discount Certificates

**Market expectations:**
- Underlying moving sideways or slightly rising
- Falling volatility
- Underlying will not breach Barrier during product lifetime

**Characteristics:**
- The underlying can be acquired at a lower price
- Cap is paid out if the Barrier is never breached
- Turns into a Discount Certificate after the Barrier is breached
- Larger probability of a maximum redemption but smaller discount can be achieved
- Lower risk than a direct investment due to the conditional capital protection
- Reduced loss potential compared to a direct investment
- Limited profit potential
Reverse Convertibles

**Market expectations:**
- Underlying trades sideways to slightly higher
- Falling volatility

**Characteristics:**
- Should the underlying close below the Strike at expiry, the underlying and/or a cash amount is redeemed
- Should the underlying close above the Strike at expiry, the nominal plus the coupon is paid at redemption.
- The coupon is always paid, irrespective of the development of the underlying
- Reduced loss potential compared to a direct investment
- Limited profit potential
- Similar to Discount Certificate
Barrier Reverse Convertible

Market expectations:
- Underlying moving sideways or slightly rising
- Falling volatility
- Underlying will not breach Barrier during product lifetime

Characteristics:
- The nominal plus coupon is paid at redemption if the Barrier never be breached
- Turns into a Reverse Convertible after breaching the barrier
- The probability of a maximum redemption is larger, but the coupon is smaller
- The coupon is always paid
- Lower risk than a direct investment due to the conditional capital protection
- Reduced loss potential compared to a direct investment
- Limited profit potential
- Similar to Barrier Discount Certificate
Capped Outperformance Certificates

**Market expectation:**
- Rising underlying

**Characteristics:**
- Reflects underlying price moves 1:1 when below the Strike
- Disproportional participation (outperformance) in a positive performance of the underlying up to the Cap
- Risk comparable to a direct investment
- Limited profit potential
Capped Bonus Certificates

**Market expectation:**
- Underlying moving sideways or slightly rising
- Underlying will not breach Barrier during product lifetime

**Characteristics:**
- Minimum redemption is equal to the Strike if the Barrier is never breached
- Lower risk than a direct investment due to the conditional capital protection
- Limited profit potential
Express Certificates

Market expectation:
- Underlying moving sideways or slightly rising
- Underlying will not breach Barrier during product lifetime

Characteristics:
- An early redemption consisting of nominal plus an additional coupon amount is paid if the underlying trade above the Strike on the observation date
- Offers the possibility of an early redemption combined with an attractive yield opportunity
- Lower risk than a direct investment due to the conditional capital protection
- Limited profit potential
Tracker Certificates

Market expectation:
- Tracker Certificate (Bull): Rising underlying
- Tracker Certificate (Bear): Falling underlying

Characteristics:
- Unlimited participation in the development of the underlying
- Reflects underlying price moves 1:1
- Risk comparable to a direct investment
Outperformance Certificates

**Market expectation:**
- Rising underlying
- Rising volatility

**Characteristics:**
- Unlimited participation in the development of the underlying
- Disproportional participation (Outperformance) in a positive performance of the underlying
- Reflects underlying price moves 1:1 when below the Strike
- The outperformance mechanism is funded by dividend payments, they are not paid out to the investor
- Risk comparable to a direct investment
Bonus Certificates

**Market expectation:**
- Underlying moving sideways or rising
- Underlying will not breach Barrier during product lifetime

**Characteristics:**
- Unlimited participation in the development of the underlying
- Turns into a Tracker Certificate after breaching the Barrier
- Minimum redemption is equal to the Strike if the Barrier is never breached
- Conditional capital protection, which is funded by dividend payments, they are not paid out to the investor
Outperformance Bonus Certificates

**Market expectation:**
- Rising underlying
- Underlying will not breach Barrier during product lifetime

**Characteristics:**
- Unlimited participation in the development of the underlying
- Minimum redemption is equal to the Strike if the Barrier is never breached
- Disproportional participation (outperformance) in a positive performance of the underlying
- Turns into an Outperformance Certificate after breaching the Barrier
- Lower risk than a direct investment due to the conditional capital protection
- Outperformance and conditional capital protection are funded by dividend payments, they are not paid out to the investor
Twin-Win Certificates

**Market expectation:**
- Rising or slightly falling underlying
- Underlying will not breach Barrier during product lifetime

**Characteristics:**
- Unlimited participation in the development of the underlying
- Minimum redemption is equal to the Strike if the Barrier is never breached
- Profits possible with rising and falling underlying
- Falling underlying price converts into profit until the Barrier
- Turns into a Tracker Certificate after breaching the Barrier
- Possible profit is funded by dividend payments, they are not paid out to the investor
Valuation with Monte Carlo Simulation

MC simulation relies on risk neutral valuation. Steps:
1) A large number of random price paths are generated for the underlying(s) with the chosen random process, which is usually geometric Brownian motion.
2) The payoff function is evaluated for each path.
3) The computed payoffs are averaged and then discounted.
Monte Carlo Simulation: Pros and Cons

**Pros:**
- Any random process can be simulated
- Correlation between underlyings can be incorporated
- Any complex payoff (e.g. any function, path dependent or conditional payoffs, early redemption) can be evaluated
- Easy to understand the concept

**Cons:**
- Computationally much more demanding than analytical or tree-based methods
- Accuracy depends on the number of simulated paths
- Especially inaccurate when calculating greeks for derivatives with discontinuous payoffs (e.g. binary options)
Making MC Simulation More Accurate

Most common variance reduction techniques:

- **Antithetic variates**: when random movements are generated for path calculation, the same movements with opposite sign are also used to calculate an additional (mirrored) path. Since these two paths are anti-correlated, the overall variance is reduced.

- **Control variates**: If there is a derivative which is both highly correlated to the simulated one and can be priced analytically, the variance of the MC result can be reduced significantly by using the difference between the analytical and the MC price of that derivative for adjustment.